

Note to the students: The answers should be written using English Language. Any books (except dictionaries), lecture notes, calculators, etc. are prohibited.

1. Solve the following differential equation. Check your answer by substitution.

$$y' = -(4 + x^2)y^3$$

2. Show that the following equation is exact and solve it. Check your answer by substitution.

$$-yx^{-2}dx + x^{-1}dy = 0$$

3. Solve the following initial value problem.

$$4y'' + 16y' + 17y = 0, \quad y(0) = -0.5, \quad y'(0) = 1$$

4. Convert the equation

$$y'' - y + y^3 = 0$$

to a (two component) system of the first order equations and study the critical points of the system.

5. Write a formal algorithm to solve a diffusion-advection-reaction system for three components $A = A(x, t)$, $B = B(x, t)$, $C = C(x, t)$, $x \in [0, 1]$ and $t \in [0, 0.03]$:

$$\dot{A} = DA'' - vA' - kAB,$$

$$\dot{B} = DB'' - vB' - kAB,$$

$$\dot{C} = DC'' - vC' + kAB,$$

with initial conditions

$$A(x, 0) = 1, \quad x \in [0, 1],$$

$$B(x, 0) = \begin{cases} 2, & x \in [0.4, 0.6], \\ 0, & \text{otherwise} \end{cases}$$

$$C(x, 0) = 0, \quad x \in [0, 1],$$

and boundary conditions

$$A'(x, t) = B'(x, t) = C'(x, t) = 0, \quad \text{at } x = 0 \text{ and } x = 1, \quad t \in [0, 0.03].$$

by the Method of Lines. The parameters of the problem are the diffusion coefficient $D = 10^{-2}$, the reaction rate $k = 10^3$ and the advection velocity $v = 10$.