LUT, Laboratory of Applied Mathematics

BM20A2100 Differential Equations

Examination 14.04.2008/ O.Shipilova

Note to the students: The answers should be written using English Language. Any books (except dictionaries), lecture notes, calculators, etc. are prohibited.

1. Solve the following differential equation. Check your answer by substitution.

$$y' = \frac{1+y}{1+x}$$

2. Show that the following equation is exact and solve it. Check your answer by substitution.

$$2xydx + x^2dy = 0$$

3. Solve the following initial value problem.

$$9y'' + 6y' + y = 0,\ y(0) = 4,\ y'(0) = -13/3$$

4. Convert the equation

$$y'' - y - y^2 = 0$$

to a (two component) system of the first order equations and study the critical points of the system.

5. Write a formal algorithm to solve a diffusion-advection-reaction system for three components $A = A(x,t)$, $B = B(x,t)$, $C = C(x,t)$, $x \in [0,1]$ and $t \in [0, 0.03]$:

$$\dot{A} = DA'' - vA' - kAB,$$
$$\dot{B} = DB'' - vB' - kAB,$$
$$\dot{C} = DC'' - vC' + kAB,$$

with initial conditions

$$A(x,0) = 1,\ x \in [0,1],$$
$$B(x,0) = \begin{cases} 2,\ x \in [0.4, 0.6], \\ 0,\ otherwise \end{cases}$$
$$C(x,0) = 0,\ x \in [0,1],$$

and boundary conditions

$$A'(x,t) = B'(x,t) = C'(x,t) = 0,\ at\ x = 0\ and\ x = 1,\ t \in [0, 0.03].$$

by the Method of Lines. The parameters of the problem are the diffusion coefficient $D = 10^{-2}$, the reaction rate $k = 10^3$ and the advection velocity $v = 10$.