

*Ti* 5416500 Statistical Analysis of Models, Examination. *MODELLING*

14.12.2005 / Heikki Haario

Note to the students: The answers may be written using either English or Finnish Language. Any books (except dictionaries), lecture notes, calculators, etc. are prohibited.

1. Derive the solution formula for the LSQ solution of the problem  $y = bx$  with data  $y_i$  at  $x_i$ ,  $i = 1, 2, \dots, n$  ('straight line through the origin'). Also give the formula for the variance of  $b$ , if  $\text{cov}(y) = \sigma^2 I$ .
2. a) How can you compute the covariance matrix of a given observation matrix?  
b) How can you produce Gaussian random vectors with a given center point and covariance matrix?
3. Describe a) the accept-reject method b) the Gibbs's algorithm
4. Describe the Metropolis MCMC algorithm. How can the information of the MCMC sample chain, produced by the algorithms, be used in estimating the reliability of modelling results?
5. Give the Bayes formula and explain the roles of the various terms in it. What is the main difficulty in using the Bayes formula, and how does the Metropolis algorithm solve it?